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Corrections to the simple effectiveness-NTU method for counterflow cooling towers and packed bed liquid desiccant–air contact systems

Cheng-Qin Ren *

College of Mechanical and Automotive Engineering, Hunan University, Changsha 410082, China

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Abstract

Analytical expressions for the corrections to the simple effectiveness-NTU method are developed using perturbation technique for cooling towers and liquid desiccant–air contact systems. The developed model takes into consideration the effect of nonlinearities of humidity ratio and enthalpy of air in equilibrium with water or desiccant solutions. The model also takes into consideration the effect of water loss by evaporation and the effect of variation of the specific heat capacity of water or solution. The comparison with numerical integration of the dimensional heat and mass transfer equations shows that the analytical results are generally satisfactory and the improvement over the simple ε-NTU method is significant.

Keywords: Cooling towers and liquid desiccant–air contact systems; Heat and mass transfer; Effectiveness-NTU method; Corrections; Perturbation technique

1. Introduction

Cooling towers and packed bed liquid desiccant–air contact systems bear much similarity in heat and mass transfer processes. Both types of these exchangers are designed to exchange heat and water species between air and liquid streams and to operate in counterflow configurations.

For simultaneous heat and mass transfer processes occurring in cooling towers, the complexity may be simplified by using the enthalpy potential proposed by Merkel [1] if Lewis factor is assumed to be unity. This assumption has been generally accepted in theoretical analyses and cooling tower design [2–6] though, for some cases, Lewis factor for moist air may be considered to possibly deviate from unity [7,8]. The Merkel method is approximate because the water film heat transfer resistance and the effect of water loss by evaporation are neglected [9]. Maclaine-cross and Banks [2] developed an analytical solution for wet surface heat exchangers by analogy from conventional solutions for dry surface heat exchangers. When the flowing fluid is water and the wall is removed, the model represents a cooling tower. The model does consider the water film heat transfer resistance but the effect of water loss by evaporation is still neglected and the humidity ratio of air in equilibrium with water surface is assumed as a linear function of water surface temperature. Jaber and Webb [3] presented an effectiveness-number of transfer units (ε-NTU) approach based on a linear variation of enthalpy of air in equilibrium with water versus temperature. The effect of nonlinearity of the equilibrium enthalpy was considered by using an enthalpy correction factor in the effectiveness definition only for the cases with smaller air flow heat capacity rate. However, the derivation of this correction factor is not convincing. The model does not consider the effect of water loss by evaporation. But if the overall heat transfer coefficient is determined from experimental data using the developed effectiveness-NTU method, then the

* Tel.: +86 13874886953; fax: +86 731 8711911.
E-mail address: renchengqin@163.com

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Nomenclature

c_p specific heat capacity, kJ/kg °C
$c_{p,\mathrm{da}}$ specific heat capacity of dry air, kJ/kg$_\mathrm{a}$ °C
$c_{p\mathrm{a}}$ specific heat capacity of moist air, kJ/kg$_\mathrm{a}$ °C
C_R ratio of an averaged saturation heat capacity rate of moist air to an averaged water or solution heat capacity rate ($\zeta_H C^*_{\mathrm{s,av}}$)
C^*_s dry air to water or solution heat capacity rate ratio ($\dot{m}_\mathrm{a} c_{p,\mathrm{da}} / \dot{m}_\mathrm{s} c_{p\mathrm{s}}$)
h specific enthalpy, kJ/kg
$h_\mathrm{c} a$ volumetric convective heat transfer coefficient, kW/m^3 °C
$h_D a_\mathrm{m}$ volumetric convective mass transfer coefficient, kg$_\mathrm{w}$/m^3 s (kg$_\mathrm{w}$/kg$_\mathrm{a}$)
$h_{\mathrm{fg},0}$ enthalpy of evaporation of water at reference temperature condition (0 °C), kJ/kg$_\mathrm{w}$
$h_{\mathrm{g,s}}$ specific enthalpy of water vapor at water or solution temperature kJ/kg$_\mathrm{w}$
$\bar{h}_{\mathrm{s,w}}$ specific enthalpy of liquid water in cooling towers or partial enthalpy of water in desiccant solution kJ/kg$_\mathrm{w}$
H dimensionless enthalpy $h/h_{\mathrm{fg},0}$
H_0, W_0 dimensionless constants in fitted equations (16) and (17) for saturation enthalpy and humidity
K coefficient in Eqs. (63) and (67)
Le_f Lewis factor
$\dot{m}$ mass flow rate, kg/s
NTU number of transfer units ($h_D aV/\dot{m}_\mathrm{a}$)
NTU$_x$ number of transfer units measured from tower top to any local position
t temperature, °C
V tower volume for heat and mass transfer, m^3
W humidity ratio of moist air, kg$_\mathrm{w}$/kg$_\mathrm{a}$
$x^{H(1)}$ correction values due to the effects of nonlinearities of the equilibrium humidity ratio and enthalpy
$x^{H(2)}$ correction values due to the effect of water loss by evaporation
x^C correction values due to the effect of variation of the heat capacity rate ratio C^*_s

Greek symbols
δh_{su} modification to the enthalpy driving potential in Eqs. (7) and (8)
δH, δW, ς_H, ς_W, σ_H, σ_C, γ constants and coefficients in Eqs. (16)–(18)
ε_0 effectiveness value for simple effectiveness-NTU method
ϑ dimensionless temperature $t \cdot c_{p,\mathrm{da}}/h_{\mathrm{fg},0}$
ξ mass fraction of desiccant in solution (wt% salt)

Subscripts
a of air
av for averaged value
i inlet
m at mean temperature $\vartheta^0_{\mathrm{s,m}} = (\vartheta_{\mathrm{s,i}} + \vartheta^0_{\mathrm{s,o}})/2$ or concentration $\xi^0_\mathrm{m} = \xi_\mathrm{i} \Big/ \left[1 + (W_{\mathrm{a,i}} - W^0_{\mathrm{a,o}}) \frac{\dot{m}_\mathrm{a}}{\dot{m}_{\mathrm{s,i}}} \Big/ 2\right]$
o outlet
s of water in cooling towers or solution in desiccant systems
ss of air in equilibrium with water or desiccant solutions
v of water vapor
w of liquid water

Superscripts
0 for zeroth order terms of perturbation expansions
H for first order terms of perturbation expansions corresponding to perturbation parameter σ_H
C for first order terms of perturbation expansions corresponding to perturbation parameter σ_C

effect of water film heat transfer resistance is actually considered. Halasz [6,10] presented a wet bulb temperature based effectiveness-NTU method for cooling towers that is somewhat similar to the Maclaine-cross and Banks' method [2]. The differences between these two models lie between the linear approximations to the equilibrium humidity ratio and between the reference conditions for evaluating the thermodynamic properties such as latent heats of evaporation and moist air specific heat capacities. Makkinejad [5] presented a mathematical solution based on linearized relationships not only between the equilibrium humidity ratio of air and the water surface temperature but also between the bulk air humidity ratio and the same water surface temperature.

Packed bed liquid desiccant–air contact systems bear much similarity to cooling towers in heat and mass transfer processes. Factor and Grossman [11] developed a one-dimensional differential heat and mass transfer model for a packed bed liquid desiccant dehumidifier/regenerator. The interface temperature and concentration were assumed to be the bulk liquid temperature and concentration. Overall heat and mass transfer coefficients were utilized. The model was validated with experimental results. This kind of model has also been utilized by other investigators to study the performances of dehumidifiers and regenerators [12,13]. Stevens et al. [13] applied the Merkel assumption, commonly used in the analysis of cooling towers, to the packed bed liquid desiccant–air contact systems. Based on linearized enthalpy temperature relationship of air in equilibrium with liquid solution, they developed an enthalpy effectiveness-NTU relation for calculating the enthalpy of air leaving the bed. An 'effective' heat and mass transfer process was assumed in which the solution stream was considered to be held at a constant 'effective' temperature that gives the correct air outlet enthalpy. With this assumption, the effective equilibrium humidity ratio and, in consequence, the outlet air humidity ratio can then be calculated.

The objective of this study is to develop analytical expressions for the corrections to the simple effectiveness-NTU method through theoretical analysis for these two types of heat exchangers. The analysis will take into consideration the effect of nonlinearities of humidity ratio and enthalpy of air in equilibrium with water or desiccant solutions. The analysis will also take into consideration the effect of water loss by evaporation and the effect of variation of the specific heat capacity of water or solution.

2. Model equations

Fig. 1 shows the schematic of cooling towers or packed bed liquid desiccant–air contact systems. On considering discussions in [7,13,14], etc., energy and mass balance equations for the heat and mass transfer processes in these exchangers can be written in a general form as

$$\dot{m}_{\mathrm{a}}\,\mathrm{d}W_{\mathrm{a}} = h_D a_{\mathrm{m}}\,\mathrm{d}V(W_{\mathrm{a,v}} - W_{\mathrm{ss}}) \tag{1}$$

$$\dot{m}_{\mathrm{a}}\,\mathrm{d}h_{\mathrm{a}} = [h_{\mathrm{c}}a(t_{\mathrm{a}} - t_{\mathrm{s}}) + h_D a_{\mathrm{m}}(W_{\mathrm{ss}} - W_{\mathrm{a,v}})h_{\mathrm{g,s}}]\,\mathrm{d}V \tag{2}$$

$$\mathrm{d}(\dot{m}_{\mathrm{s}}h_{\mathrm{s}}) = \dot{m}_{\mathrm{a}}\,\mathrm{d}h_{\mathrm{a}} \tag{3}$$

Here, $W_{\mathrm{a,v}}$ represents the moisture content of air in vapor state. For unsaturated or saturated air, $W_{\mathrm{a,v}}$ will be just

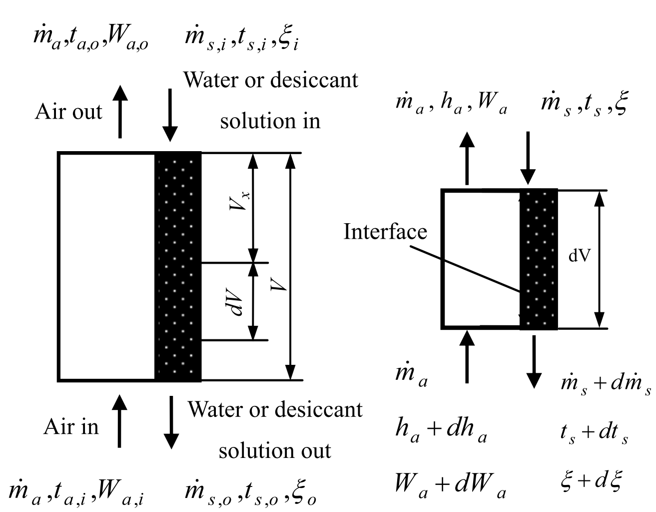

Fig. 1. Schematic of cooling towers or packed bed liquid desiccant–air contact systems.

equal to the humidity ratio of air W_{a}. If supersaturated conditions occur, $W_{\mathrm{a,v}}$ will be equal to the saturation humidity ratio W_{sa} corresponding to the temperature t_{a}. In general, the specific enthalpy of moist air can be calculated as [14]

$$h_{\mathrm{a}} = c_{pa}t_{\mathrm{a}} + W_{\mathrm{a,v}}h_{\mathrm{fg,0}} \tag{4}$$

Here, $c_{pa} = c_{p,\mathrm{da}} + W_{\mathrm{a,v}}c_{pv} + (W_{\mathrm{a}} - W_{\mathrm{a,v}})c_{p\mathrm{w}}$. For water or desiccant solutions, the Gibbs equation is

$$\mathrm{d}(\dot{m}_{\mathrm{s}}h_{\mathrm{s}}) = \dot{m}_{\mathrm{s}}c_{ps}\,\mathrm{d}t_{\mathrm{s}} + \bar{h}_{\mathrm{s,w}}\,\mathrm{d}\dot{m}_{\mathrm{s}} \tag{5}$$

Following some conventional practices [3,13] in modeling cooling towers and packed bed desiccant systems, Lewis relation is adopted

$$Le_{\mathrm{f}} = h_{\mathrm{c}}a/(h_D a_{\mathrm{m}} c_{pa}) = 1 \tag{6}$$

Combining above equations can give rearranged energy equations as follows:

$$\mathrm{d}h_{\mathrm{a}} = (h_{\mathrm{a}} - h_{\mathrm{ss}} - \delta h_{\mathrm{su}})\,\mathrm{dNTU}_x \tag{7}$$

$$\mathrm{d}t_{\mathrm{s}} = \frac{\dot{m}_{\mathrm{a}}}{\dot{m}_{\mathrm{s}}c_{ps}}\left[(h_{\mathrm{a}} - h_{\mathrm{ss}} - \delta h_{\mathrm{su}}) - \bar{h}_{\mathrm{s,w}}(W_{\mathrm{a,v}} - W_{\mathrm{ss}})\right]\mathrm{dNTU}_x \tag{8}$$

Here, $\delta h_{\mathrm{su}} = (W_{\mathrm{a}} - W_{\mathrm{a,v}})c_{p\mathrm{w}}t_{\mathrm{s}}$ is a modification to the enthalpy driving potential to correct for the effect of supersaturation in air. For liquid desiccant system, supersaturation will not occur and δh_{su} is equal to zero. For cooling towers, supersaturation may possibly occur. But even if this does occur, the effect is found to be very small through numerical experiments. Thus, for simplicity in the process of development of analytical solutions, δh_{su} is considered to be zero, $W_{\mathrm{a,v}}$ is taken to be equal to W_{a} and specific heat capacity of moist air is calculated as $c_{pa} = c_{p,\mathrm{da}} + W_{\mathrm{a}}c_{pv}$, as if there is no saturation restriction to the moisture content in air. This, however, does not mean that no supersaturation effect can be considered in analytical solutions. If supersaturation does occur, final results of humidity ratio and enthalpy of air are assumed to be unaffected but temperature t_{a} should be calculated according to Eq. (4) with actual c_{pa} and $W_{\mathrm{a,v}}$ values. For conformity, Eq. (1) is also rearranged and is given as follows:

$$\mathrm{d}W_{\mathrm{a}} = (W_{\mathrm{a,v}} - W_{\mathrm{ss}})\,\mathrm{dNTU}_x \tag{9}$$

Dimensionless enthalpy and temperature are defined as follows:

$$H = h/h_{\mathrm{fg,0}} \tag{10}$$

$$\vartheta = t \cdot c_{p,\mathrm{da}}/h_{\mathrm{fg,0}} \tag{11}$$

Transforming Eqs. (7) and (8) by definitions of Eqs. (10) and (11) to dimensionless form and neglecting the saturation restriction to the moisture content in air will give equations as

$$\mathrm{d}H_{\mathrm{a}} = (H_{\mathrm{a}} - H_{\mathrm{ss}})\,\mathrm{dNTU}_x \tag{12}$$

$$\mathrm{d}\vartheta_{\mathrm{s}} = C_{\mathrm{s}}^{*}[(H_{\mathrm{a}} - H_{\mathrm{ss}}) - \overline{H}_{\mathrm{s,w}}(W_{\mathrm{a}} - W_{\mathrm{ss}})]\,\mathrm{dNTU}_x \tag{13}$$

Here, $\overline{H}_{\rm s,w} = \bar{h}_{\rm s,w}/h_{\rm fg,0}$. Also by neglecting the saturation restriction to the moisture content in air, Eq. (9) can be rewritten as follows:

$$\mathrm{d}W_{\rm a} = (W_{\rm a} - W_{\rm ss})\,\mathrm{dNTU}_x \tag{14}$$

For definite solutions, Eqs. (12)–(14) should be solved under the following boundary conditions

$$\begin{aligned} &\mathrm{NTU}_x = 0, \quad \vartheta_{\rm s} = \vartheta_{\rm s,i} = t_{\rm s,i} c_{p,\rm da}/h_{\rm fg,0} \\ &\mathrm{NTU}_x = \mathrm{NTU}, \quad H_{\rm a} = H_{\rm a,i} = h_{\rm a,i}/h_{\rm fg,0}, \quad W_{\rm a} = W_{\rm a,i} \end{aligned} \tag{15}$$

3. Perturbation method

Due to the nonlinearities of humidity ratio and enthalpy of air in equilibrium with water or desiccant solutions, it is hard to get an exact analytical solution for the above equations. Approximate analytical solutions, however, can be obtained by using the perturbation method. The equilibrium humidity ratio and enthalpy of air are then fitted with following equations:

$$H_{\rm ss} = H_0 + \varsigma_H \vartheta_{\rm s} + \sigma_H[(\vartheta_{\rm s} - \vartheta_{\rm s,i})(\vartheta_{\rm s} - \vartheta^0_{\rm s,o}) + \delta_H] \tag{16}$$

$$W_{\rm ss} = W_0 + \varsigma_W \vartheta_{\rm s} + \sigma_H \gamma[(\vartheta_{\rm s} - \vartheta_{\rm s,i})(\vartheta_{\rm s} - \vartheta^0_{\rm s,o}) + \delta_W] \tag{17}$$

Liquid specific heat capacity often changes with its temperature and thus the heat capacity rate ratio $C^*_{\rm s}$ is fitted as

$$C^*_{\rm s} = C^*_{\rm s,av} + \sigma_C(\vartheta_{\rm s} - \vartheta^0_{\rm s,m}) \tag{18}$$

Here $\vartheta^0_{\rm s,m} = (\vartheta_{\rm s,i} + \vartheta^0_{\rm s,o})/2$. In expanding Eqs. (12)–(15), σ_H and σ_C are used as the perturbation parameters. Solution of Eqs. (12)–(15) can then be expressed approximately by the perturbation expansions up to the first order as follows:

$$H_{\rm a} = H^0_{\rm a} + \sigma_H H^H_{\rm a} + \sigma_C H^C_{\rm a} \tag{19}$$

$$\vartheta_{\rm s} = \vartheta^0_{\rm s} + \sigma_H \vartheta^H_{\rm s} + \sigma_C \vartheta^C_{\rm s} \tag{20}$$

$$W_{\rm a} = W^0_{\rm a} + \sigma_H W^H_{\rm a} + \sigma_C W^C_{\rm a} \tag{21}$$

Substituting Eqs. (16)–(18) into Eqs. (12)–(14) and expanding the resultant equations with Eqs. (19)–(21) will give the following equations:

$$\mathrm{d}H^0_{\rm a} = (H^0_{\rm a} - \varsigma_H \vartheta^0_{\rm s} - H_0)\,\mathrm{dNTU}_x \tag{22}$$

$$\mathrm{d}\vartheta^0_{\rm s} = C^*_{\rm s,av}(H^0_{\rm a} - \varsigma_H \vartheta^0_{\rm s} - H_0)\,\mathrm{dNTU}_x \tag{23}$$

$$\mathrm{d}W^0_{\rm a} = (W^0_{\rm a} - \varsigma_W \vartheta^0_{\rm s} - W_0)\,\mathrm{dNTU}_x \tag{24}$$

$$\mathrm{d}H^H_{\rm a} = \left[H^H_{\rm a} - \varsigma_H \vartheta^H_{\rm s} - (\vartheta^0_{\rm s} - \vartheta_{\rm s,i})(\vartheta^0_{\rm s} - \vartheta^0_{\rm s,o}) - \delta_H\right]\mathrm{dNTU}_x \tag{25}$$

$$\begin{aligned} \mathrm{d}\vartheta^H_{\rm s} = C^*_{\rm s,av}\Big[&H^H_{\rm a} - \varsigma_H \vartheta^H_{\rm s} - (\vartheta^0_{\rm s} - \vartheta_{\rm s,i})(\vartheta^0_{\rm s} - \vartheta^0_{\rm s,o}) - \delta_H \\ &- \overline{H}_{\rm s,w}(W^0_{\rm a} - W^0_{\rm ss})/\sigma_H\Big]\mathrm{dNTU}_x \end{aligned} \tag{26}$$

$$\mathrm{d}W^H_{\rm a} = \left[W^H_{\rm a} - \varsigma_W \vartheta^H_{\rm s} - \gamma(\vartheta^0_{\rm s} - \vartheta_{\rm s,i})(\vartheta^0_{\rm s} - \vartheta^0_{\rm s,o}) - \delta_W\right]\mathrm{dNTU}_x \tag{27}$$

$$\mathrm{d}H^C_{\rm a} = (H^C_{\rm a} - \varsigma_H \vartheta^C_{\rm s})\,\mathrm{dNTU}_x \tag{28}$$

$$\mathrm{d}\vartheta^C_{\rm s} = \left[C^*_{\rm s,av}(H^C_{\rm a} - \varsigma_H \vartheta^C_{\rm s}) + (\vartheta^0_{\rm s} - \vartheta^0_{\rm s,av})(H^0_{\rm a} - \varsigma_H \vartheta^0_{\rm s} - H_0)\right]\mathrm{dNTU}_x \tag{29}$$

$$\mathrm{d}W^C_{\rm a} = (W^C_{\rm a} - \varsigma_W \vartheta^C_{\rm s})\,\mathrm{dNTU}_x \tag{30}$$

Expanding Eq. (15) with Eqs. (19)–(21) will give the boundary conditions as follows:

$$\begin{aligned} &\mathrm{NTU}_x = 0, \quad \vartheta^0_{\rm s} = \vartheta_{\rm s,i} = t_{\rm s,i} c_{p,\rm da}/h_{\rm fg,0}, \quad \vartheta^H_{\rm s} = \vartheta^C_{\rm s} = 0 \\ &\mathrm{NTU}_x = \mathrm{NTU}, \quad H^0_{\rm a} = H_{\rm a,i} = h_{\rm a,i}/h_{\rm fg,0}, \quad W^0_{\rm a} = W_{\rm a,i}, \\ &H^H_{\rm a} = W^H_{\rm a} = H^C_{\rm a} = W^C_{\rm a} = 0 \end{aligned} \tag{31}$$

When desiccant concentrations are to be calculated, the following equation will apply:

$$\begin{aligned} &\xi = \xi_{\rm i} \Big/ \left[1 + (W_{\rm a} - W_{\rm a,o})\frac{\dot{m}_{\rm a}}{\dot{m}_{\rm s,i}}\right] \quad \text{or} \\ &\xi_{\rm o} = \xi_{\rm i} \Big/ \left[1 + (W_{\rm a,i} - W_{\rm a,o})\frac{\dot{m}_{\rm a}}{\dot{m}_{\rm s,i}}\right] \end{aligned} \tag{32}$$

4. Simple model correlations

Simple model correlations are those to be developed from the analysis of the zeroth order equations. Noting that $\varsigma_H \vartheta^0_{\rm s} + H_0$ is, in zeroth order approximation, the enthalpy of air in equilibrium with water or desiccant solution $H^0_{\rm ss}$, Eqs. (22) and (23) can be rewritten as

$$\mathrm{d}H^0_{\rm a} = (H^0_{\rm a} - H^0_{\rm ss})\,\mathrm{dNTU}_x \tag{33}$$

$$\mathrm{d}H^0_{\rm ss} = C_{\rm R}(H^0_{\rm a} - H^0_{\rm ss})\,\mathrm{dNTU}_x \tag{34}$$

Using enthalpy driving potential, these two equations combined with the boundary condition equation (31) are actually similar to the model for conventional counterflow heat exchangers. Thus, solution for these equations can be obtained just by using the simple effectiveness-NTU method.

$$H^0_{\rm a,o} = H_{\rm a,i} + \varepsilon_0(H^0_{\rm ss,i} - H_{\rm a,i}) \tag{35}$$

$$\vartheta^0_{\rm s,o} = \vartheta_{\rm s,i} - C^*_{\rm s,av}\varepsilon_0(H^0_{\rm ss,i} - H_{\rm a,i}) \tag{36}$$

Here, $H^0_{\rm ss,i} = H_0 + \varsigma_H \vartheta_{\rm s,i}$. The effectiveness ε_0 is correlated to NTU by the simple effectiveness-NTU relation as for the conventional counterflow heat exchangers

$$\varepsilon_0 = \frac{\mathrm{e}^{(1-C_{\rm R})\mathrm{NTU}} - 1}{\mathrm{e}^{(1-C_{\rm R})\mathrm{NTU}} - C_{\rm R}} \quad \text{or} \quad \mathrm{e}^{(1-C_{\rm R})\mathrm{NTU}} = \frac{1 - C_{\rm R}\varepsilon_0}{1 - \varepsilon_0} \tag{37}$$

with $C_{\rm R} = \varsigma_H C^*_{\rm s,av}$. For cooling towers, this simplified model bears much similarity with those developed by Jaber and Webb [3] and by Halasz [6,10]. In Halasz [6,10], wet bulb temperature effectiveness is utilized. Effectiveness values calculated by these models will be equal if the total heat capacity rate ratios z in Halasz [6,10] and $C_{\rm R}$'s in Jaber and Webb [3] and in this paper are equal. This, however, will only be approximately satisfied due to the differences in the linearization of air enthalpy against wet bulb temperature and/or in the linearization of the equilibrium enthalpy of air against air–water interface temperature. For liquid desiccant systems, this result is equivalent to that by Stevens et al. [13] provided that C_R is equivalent to their m^* value and the linearizations of the enthalpy of air in equilibrium with desiccant solutions are the same.

Further, combining Eqs. (22) and (23) gives the following equation:

$$d(H_a^0 - \varsigma_H \vartheta_s^0 - H_0) = (1 - C_R)(H_a^0 - \varsigma_H \vartheta_s^0 - H_0)\,d\mathrm{NTU}_x \tag{38}$$

Integrating Eq. (38) under the boundary condition equation (31) gives

$$(H_a^0 - \varsigma_H \vartheta_s^0 - H_0) = (H_{a,o}^0 - H_{ss,i}^0)e^{(1-C_R)\mathrm{NTU}_x} \tag{39}$$

Substituting Eq. (39) into Eq. (23) and integrating the resultant equation under the same boundary condition give the following equation:

$$\vartheta_s^0 = \frac{C_{s,av}^*}{1 - C_R}(H_{a,o}^0 - H_{ss,i}^0)\left(e^{(1-C_R)\mathrm{NTU}_x} - 1\right) + \vartheta_{s,i} \tag{40}$$

Using Eq. (35) to eliminate $H_{a,o}^0$ in Eq. (40) gives

$$\vartheta_s^0 = \vartheta_{s,i} - C_{s,av}^* \frac{1 - \varepsilon_0}{1 - C_R}(H_{ss,i}^0 - H_{a,i})\left(e^{(1-C_R)\mathrm{NTU}_x} - 1\right) \tag{41}$$

And using Eqs. (35) and (41) to eliminate $H_{a,o}^0$ and ϑ_s^0 in Eq. (39) gives

$$H_a^0 = H_{ss,i}^0 - \frac{1 - \varepsilon_0}{1 - C_R}(H_{ss,i}^0 - H_{a,i})\left(e^{(1-C_R)\mathrm{NTU}_x} - C_R\right) \tag{42}$$

Substituting Eq. (41) into Eq. (24) and integrating the resultant equation under the boundary condition equation (31) will give the expression for air humidity ratio as

$$\begin{aligned} W_a^0 = {} & W_{ss,i}^0 - (W_{ss,i}^0 - W_{a,i})e^{\mathrm{NTU}_x - \mathrm{NTU}} \\ & - C_1(H_{ss,i}^0 - H_{a,i})\left[\left(e^{\mathrm{NTU}_x - \mathrm{NTU}} - 1\right) \right. \\ & \left. - \left(e^{\mathrm{NTU}_x - C_R\mathrm{NTU}} - e^{(1-C_R)\mathrm{NTU}_x}\right)/C_R\right] \end{aligned} \tag{43}$$

Let $\mathrm{NTU}_x = 0$ in Eq. (43) to give the outlet value as

$$\begin{aligned} W_{a,o}^0 = {} & W_{ss,i}^0 - (W_{ss,i}^0 - W_{a,i})e^{-\mathrm{NTU}} \\ & - C_2(H_{ss,i}^0 - H_{a,i})\left(1 - \frac{1}{1 - \varepsilon_0}e^{-\mathrm{NTU}}\right) \end{aligned} \tag{44}$$

In Eqs. (43) and (44), $W_{ss,i}^0 = W_0 + \varsigma_W \vartheta_{s,i}$, $C_1 = \varsigma_W C_{s,av}^*(1 - \varepsilon_0)/(1 - C_R)$ and $C_2 = \varsigma_W C_{s,av}^*(1 - \varepsilon_0)/C_R$. Eqs. (41)–(44) are a supplement to the simple effectiveness-NTU method and can be utilized to calculate outlet air humidity ratio as well as parameter profiles along the tower height.

5. Correction correlations

Simple model correlations neglect the effects of nonlinearities of humidity ratio and enthalpy of air in equilibrium with water or desiccant solutions and the effect of water loss by evaporation. They also neglect the effect of variation of the heat capacity rate ratio C_s^*. Thus, results determined by these correlations need to be corrected for an improved accuracy in rating and/or design calculations. The second and third terms on the right-hand side of Eqs. (19)–(21) are the corrections to the results of zeroth order equations. Analysis of first order equations will provide the correlations for determining these correction values.

5.1. For the first set of correction terms

Correlations for H_a^H, ϑ_s^H and W_a^H are in relation with the first set of correction terms. These correlations will be derived by integrating Eqs. (25)–(27).

From Eqs. (25) and (26), the following equation can be derived:

$$\begin{aligned} d(H_a^H - \varsigma_H \vartheta_s^H) = (1 - C_R)\Big[& H_a^H - \varsigma_H \vartheta_s^H - (\vartheta_s^0 - \vartheta_{s,i})(\vartheta_s^0 - \vartheta_{s,o}^0) \\ & - \delta_H + \frac{C_R}{1 - C_R}\overline{H}_{s,w}(W_a^0 - W_{ss}^0)/\sigma_H\Big] d\mathrm{NTU}_x \end{aligned} \tag{45}$$

Here $W_{ss}^0 = W_0 + \varsigma_W \vartheta_s^0$. Taking $\overline{H}_{s,w}$ to be constant as its average value $(\overline{H}_{s,w})_{av}$, substituting Eqs. (41) and (43) into Eq. (45) and integrating the resultant equation lead to the following equation:

$$\begin{aligned} H_a^H - \varsigma_H \vartheta_s^H = {} & C_3 \frac{e^{2(1-C_R)\mathrm{NTU}_x}}{1 - C_R} + (C_4\mathrm{NTU}_x + C_7)e^{(1-C_R)\mathrm{NTU}_x} \\ & + \frac{C_5}{C_R}e^{\mathrm{NTU}_x - \mathrm{NTU}} - \frac{C_6}{1 - C_R} \end{aligned} \tag{46}$$

Here

$$C_3 = -\frac{C_{s,av}^{*2}(1 - \varepsilon_0)^2(H_{ss,i}^0 - H_{a,i})^2}{1 - C_R}$$

$$C_4 = -C_3 \frac{2 - \varepsilon_0 - C_R\varepsilon_0}{1 - \varepsilon_0} - \frac{(\overline{H}_{s,w})_{av}}{\sigma_H}\varsigma_W C_{s,av}^*(1 - \varepsilon_0)(H_{ss,i}^0 - H_{a,i})$$

$$C_5 = C_R \frac{(\overline{H}_{s,w})_{av}}{\sigma_H}\left[-(W_{ss,i}^0 - W_{a,i}) + \frac{C_2(H_{ss,i}^0 - H_{a,i})}{1 - \varepsilon_0}\right]$$

$$C_6 = C_3 \frac{1 - C_R\varepsilon_0}{1 - \varepsilon_0} - \delta_H(1 - C_R)$$

Substituting Eqs. (41) and (46) into Eq. (25) and integrating the resultant equation give the equation

$$\begin{aligned} H_a^H = {} & \frac{C_3}{(1 - C_R)^2}e^{2(1-C_R)\mathrm{NTU}_x} + \frac{1}{1 - C_R}(C_4\mathrm{NTU}_x \\ & + C_7 - C_8)e^{(1-C_R)\mathrm{NTU}_x} + \frac{C_5}{C_R}e^{\mathrm{NTU}_x - \mathrm{NTU}} + C_9 \end{aligned} \tag{47}$$

Here $C_8 = -\frac{(\overline{H}_{s,w})_{av} C_R}{\sigma_H(1 - C_R)} C_2(H_{ss,i}^0 - H_{a,i})$. Substituting Eq. (47) into Eq. (46) gives

$$\begin{aligned} \vartheta_s^H = \Big\{ & \frac{C_3 C_R}{(1 - C_R)^2}e^{2(1-C_R)\mathrm{NTU}_x} \\ & + \left(\frac{C_4 C_R \mathrm{NTU}_x + C_7 C_R - C_8}{1 - C_R}\right)e^{(1-C_R)\mathrm{NTU}_x} \\ & + C_9 + \frac{C_6}{1 - C_R}\Big\} \Big/ \varsigma_H \end{aligned} \tag{48}$$

Satisfying Eqs. (47) and (48) to the boundary condition equation (31) gives the integration constants C_7 and C_9 as

$$C_7 = C_3\frac{\varepsilon_0(2C_R\varepsilon_0 - 1 - C_R)}{(1-C_R)(1-\varepsilon_0)} + C_8\varepsilon_0 - C_4\frac{1-C_R\varepsilon_0}{1-C_R}\mathrm{NTU} - C_5\frac{1-\varepsilon_0}{C_R} - \delta_H(1-\varepsilon_0)$$

$$C_9 = \left[-C_3\frac{1-2C_R\varepsilon_0}{(1-\varepsilon_0)(1-C_R)} + C_4\frac{C_R}{1-C_R}\mathrm{NTU} + C_8\right]\frac{1-C_R\varepsilon_0}{1-C_R} + C_5\frac{1-\varepsilon_0}{1-C_R} + \frac{1-C_R\varepsilon_0}{1-C_R}\delta_H$$

For outlet positions, we have

$$H^H_{a,o} = \frac{C_3}{(1-C_R)^2} + \frac{C_5}{C_R}e^{-\mathrm{NTU}} + \frac{C_7 - C_8}{1-C_R} + C_9 \tag{49}$$

$$\vartheta^H_{s,o} = \left\{(1-C_R\varepsilon_0)\left[\frac{C_R}{1-C_R}\left(\frac{2\varepsilon_0}{1-\varepsilon_0}C_3 + C_4\mathrm{NTU}\right) - \frac{\varepsilon_0}{1-\varepsilon_0}C_8\right] - \varepsilon_0 C_5 - C_R\varepsilon_0\delta_H\right\}\Big/\varsigma_H \tag{50}$$

Substituting Eqs. (41) and (47) into Eq. (27) and integrating the resultant equation give the following equation:

$$W^H_a = C_{11}e^{2(1-C_R)\mathrm{NTU}_x} + (C_{12}\mathrm{NTU}_x + C_{13})e^{(1-C_R)\mathrm{NTU}_x} + C_{14} + C_{15}e^{\mathrm{NTU}_x} \tag{51}$$

Here, constants C_{10}–C_{14} are as follows:

$$C_{10} = -\frac{C_3}{1-C_R}\,\frac{2-\varepsilon_0 - C_R\varepsilon_0}{1-\varepsilon_0}$$

$$C_{11} = \frac{C_3}{1-2C_R}\left(\frac{\gamma}{1-C_R} - \frac{\varsigma_W C_R}{\varsigma_H(1-C_R)^2}\right)$$

$$C_{12} = \frac{\varsigma_W}{\varsigma_H(1-C_R)}C_4$$

$$C_{13} = \frac{1}{C_R}\left(\frac{\varsigma_W}{\varsigma_H}\,\frac{C_4 + C_R C_7 - C_8}{1-C_R} - \gamma C_{10}\right)$$

$$C_{14} = \frac{\varsigma_W}{\varsigma_H}C_9 - \frac{C_6}{1-C_R}\left(\gamma - \frac{\varsigma_W}{\varsigma_H}\right) + \delta_W - \gamma\delta_H$$

Satisfying Eq. (51) to the boundary condition equation (31) gives the integration constant

$$C_{15} = -e^{-\mathrm{NTU}}\left\{\left(C_{11}\frac{1-C_R\varepsilon_0}{1-\varepsilon_0} + C_{12}\mathrm{NTU} + C_{13}\right)\frac{1-C_R\varepsilon_0}{1-\varepsilon_0} + C_{14}\right\}$$

Let $\mathrm{NTU}_x = 0$ to give

$$W^H_{a,o} = C_{11} + C_{13} + C_{14} + C_{15} \tag{52}$$

In above correlations, each term of H^H_a, ϑ^H_s and W^H_a can be logically divided into two parts. The parts proportional to $(\overline{H}_{s,w})_{av}$ in the expressions of H^H_a, ϑ^H_s and W^H_a are caused by the effect of water loss by evaporation on water or solution energy balance. The remaining parts are due to the effects of nonlinearities of the humidity ratio and enthalpy of air in equilibrium with the water or desiccant solution. These can be obtained by setting $(\overline{H}_{s,w})_{av} = 0$ in calculations.

5.2. For the second set of correction terms

Correlations for H^C_a, ϑ^C_s and W^C_a are in relation with the second set of correction terms for the effect of variation of the heat capacity rate ratio C^*_s. These correlations will be derived by integrating Eqs. (28)–(30).

From Eqs. (28) and (29), the following equation can be derived:

$$d(H^C_a - \varsigma_H\vartheta^C_s) = \Big[(1-C_R)(H^C_a - \varsigma_H\vartheta^C_s) - \varsigma_H(\vartheta^0_s - \vartheta^0_{s,m})(H^0_a - \varsigma_H\vartheta^0_s - H_0)\Big]d\mathrm{NTU}_x \tag{53}$$

Substitute Eqs. (41) and (42) into Eq. (53) and integrate the resultant equation to give

$$H^C_a - \varsigma_H\vartheta^C_s = \frac{C_3\varsigma_H}{(1-C_R)C^*_{s,av}}e^{2(1-C_R)\mathrm{NTU}_x} + \left(\frac{C_{10}(1-C_R)\varsigma_H}{2C^*_{s,av}}\mathrm{NTU}_x + C_{16}\right)e^{(1-C_R)\mathrm{NTU}_x} \tag{54}$$

Further substitute Eq. (54) into Eq. (28) and integrate the resultant equation to give

$$H^C_a = \frac{\varsigma_H C_3}{2C^*_{s,av}}\,\frac{e^{2(1-C_R)\mathrm{NTU}_x}}{(1-C_R)^2} + \left[\frac{C_{10}(1-C_R)\varsigma_H}{2C^*_{s,av}}\left(\mathrm{NTU}_x - \frac{1}{1-C_R}\right) + C_{16}\right]\frac{e^{(1-C_R)\mathrm{NTU}_x}}{1-C_R} + C_{17} \tag{55}$$

Again substitute Eq. (55) into Eq. (54) to give

$$\theta^C_s = -C_3\frac{1-2C_R}{2C^*_{s,av}}\,\frac{e^{2(1-C_R)\mathrm{NTU}_x}}{(1-C_R)^2} + \left[\frac{C_{10}(1-C_R)}{2C^*_{s,av}}\left(C_R\mathrm{NTU}_x - \frac{1}{1-C_R}\right) + C_{16}C^*_{s,av}\right]\frac{e^{(1-C_R)\mathrm{NTU}_x}}{1-C_R} + C_{17}/\varsigma_H \tag{56}$$

Satisfying Eqs. (55) and (56) to the boundary condition equation (31) gives the integration constants as

$$C_{16} = -\frac{\varsigma_H}{C^*_{s,av}}\left[\frac{C_{10}}{2}\mathrm{NTU}(1-C_R\varepsilon_0) + C_3\frac{1-C_R\varepsilon_0^2}{(1-\varepsilon_0)(1-C_R)}\right]$$

$$C_{17} = \varsigma_H\frac{(1-2C_R)C_3 + C_{10}(1-C_R)}{2C^*_{s,av}(1-C_R)^2} - \frac{C_{16}C_R}{1-C_R}$$

Let $\mathrm{NTU}_x = 0$ in Eq. (55) to give

$$H^C_{a,o} = \frac{\varsigma_H C_3}{C^*_{s,av}(1-C_R)} + C_{16} \tag{57}$$

Let $\mathrm{NTU}_x = \mathrm{NTU}$ in Eq. (56) to give

$$\theta^C_{\mathrm{s,o}} = -C^*_{\mathrm{s,av}} H^C_{\mathrm{a,o}} \tag{58}$$

Substituting Eq. (56) into Eq. (30) and integrating the resultant equation under the boundary condition equation (31) give

$$W^C_{\mathrm{a}} = \varsigma_W \frac{\mathrm{e}^{(1-C_\mathrm{R})\mathrm{NTU}_x}}{1-C_\mathrm{R}} \left\{ \frac{1}{2C^*_{\mathrm{s,av}}} \left[\frac{C_3 \mathrm{e}^{(1-C_\mathrm{R})\mathrm{NTU}_x}}{1-C_\mathrm{R}} + C_{10}(1-C_\mathrm{R}) \left(\mathrm{NTU}_x - \frac{1}{1-C_\mathrm{R}} \right) \right] + \frac{C_{16}}{\varsigma_H} \right\} + \frac{C_{17}}{\varsigma_H} \varsigma_W \tag{59}$$

For the outlet position, $\mathrm{NTU}_x = 0$ and

$$W^C_{\mathrm{a,o}} = \frac{\varsigma_W C_3}{C^*_{\mathrm{s,av}}(1-C_\mathrm{R})} + \frac{C_{16}}{\varsigma_H} \varsigma_W \tag{60}$$

6. Comparison and discussion

Through above discussion, supplements and corrections to the simple effectiveness-NTU method for cooling towers and packed bed liquid desiccant–air contact systems are developed using perturbation method. In order to demonstrate the validity of the developed method, results are compared with those from numerical integrations of Eqs. (7)–(9).

To perform calculation with the developed method, constants and coefficients in the fitted thermodynamic property equations (16)–(18) will be evaluated at first. They are calculated according to the following equations:

$$\varsigma_H = (H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}}) - H_{\mathrm{ss,i}})/(\vartheta^0_{\mathrm{s,o}} - \vartheta_{\mathrm{s,i}}) \tag{61}$$

$$\sigma_H = 4\{[H_{\mathrm{ss,i}} + H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}})]/2 - H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})\}/(\vartheta^0_{\mathrm{s,o}} - \vartheta_{\mathrm{s,i}})^2 \tag{62}$$

$$\delta_H = K(H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}}) + H_{\mathrm{ss,i}} - 2H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}}))/(3\sigma_H) \tag{63}$$

$$H_0 = H_{\mathrm{ss,i}} - \sigma_H \delta_H - \varsigma_H \vartheta_{\mathrm{s,i}} \tag{64}$$

$$\varsigma_W = (W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}}) - W_{\mathrm{ss,i}})/(\vartheta^0_{\mathrm{s,o}} - \vartheta_{\mathrm{s,i}}) \tag{65}$$

$$\gamma = 4\{[W_{\mathrm{ss,i}} + W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}})]/2 - W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})\}/[(\vartheta^0_{\mathrm{s,o}} - \vartheta_{\mathrm{s,i}})^2 \sigma_H] \tag{66}$$

$$\delta_W = K(W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}}) + W_{\mathrm{ss,i}} - 2W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}}))/(3\gamma\sigma_H) \tag{67}$$

$$W_0 = W_{\mathrm{ss,i}} - \gamma\sigma_H \delta_W - \varsigma_W \vartheta_{\mathrm{s,i}} \tag{68}$$

$$C^*_{\mathrm{s,av}} = (C^*_{\mathrm{s}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}}) + C^*_{\mathrm{s,i}} + 4C^*_{\mathrm{s}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}}))/6 \tag{69}$$

$$\sigma_C = (C^*_{\mathrm{s,o}} - C^*_{\mathrm{s,i}})/(\vartheta^0_{\mathrm{s,o}} - \vartheta_{\mathrm{s,i}}) \tag{70}$$

In above equations, $H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})$, $H_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}})$, $W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})$, $W_{\mathrm{ss}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}})$, $C^*_{\mathrm{s}}(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})$ and $C^*_{\mathrm{s}}(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}})$ represent real thermodynamic property values H_{ss}, W_{ss} and C^*_{s} at states $(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})$ and $(\vartheta^0_{\mathrm{s,o}}, \xi^0_{\mathrm{o}})$, respectively. Parameters σ_H and γ are evaluated such that the fitted equations (16) and (17) give the actual equilibrium enthalpy and humidity values at state $(\vartheta^0_{\mathrm{s,m}}, \xi^0_{\mathrm{m}})$. K is a set value. For comparison, K is set to unity for cooling towers and set to zero for packed bed liquid desiccant–air contact systems. The zeroth order approximation of saturation humidity by Eq. (17) with $K = 1$ for cooling towers is identical to the linear approximation proposed by Maclaine-cross and Banks [2], which is said to give an approximate least squares fit to the true or actual saturation line over the range of water surface temperatures and is considered to give more accurate water outlet conditions. When K is set to zero for liquid desiccant systems, the zeroth order approximation of the equilibrium enthalpy by Eq. (16) is identical to the linear approximation adopted by Stevens et al. [13].

Still, two general steps are needed. Firstly, one has to perform calculation using the simple model correlations developed by the analysis of the zeroth order equations. Then, one has to calculate corrections using the resultant correlations developed by the analysis of the first order equations. From Eqs. (61)–(70), we can see that the correct evaluation of those constants and coefficients will also depend on the calculated values of outlet parameters $\vartheta^0_{\mathrm{s,o}}$ and ξ^0_{o}. Thus, iterations are needed and the steps are as follows:

1. Determine dimensionless values $\vartheta_{\mathrm{s,i}}$, $H_{\mathrm{ss,i}}$, $H_{\mathrm{a,i}}$, $W_{\mathrm{a,i}}$ and NTU from given conditions and properties of water or desiccant solutions [15–17].
2. Use Eqs. (61)–(70) to evaluate constants and coefficients ς_H, σ_H, δ_H, H_0, ς_W, γ, δ_W, W_0, $C^*_{\mathrm{s,av}}$ and σ_C based on assumed or calculated outlet parameters $\vartheta^0_{\mathrm{s,o}}$ and ξ^0_{o}.
3. Use Eqs. (35)–(37) and Eq. (44) to calculate effectiveness value ε_0 and outlet parameters $H^0_{\mathrm{a,o}}$, $\vartheta^0_{\mathrm{s,o}}$ and $W^0_{\mathrm{a,o}}$ and use Eq. (32) to calculate outlet concentration ξ^0_{o}.
4. Repeat steps 2 and 3 until the calculated outlet parameters matched the assumed values.
   Further steps are needed for calculating the corrections and, in consequence, the final results of outlet parameters. These steps are given as follows:
5. Calculate constants C_1–C_{17} using their corresponding expressions.
6. Calculate quantities H^H_{a}, ϑ^H_{s}, W^H_{a}, H^C_{a}, ϑ^C_{s} and W^C_{a} using correlations developed by analysis of first order equations in Section 5.
7. Use Eqs. (19)–(21) to calculate outlet parameters $H_{\mathrm{a,o}}$, $\vartheta_{\mathrm{s,o}}$ and $W_{\mathrm{a,o}}$, then use Eqs. (10) and (11) to transform $H_{\mathrm{a,o}}$ and $\vartheta_{\mathrm{s,o}}$ to dimensional values $h_{\mathrm{a,o}}$ and $t_{\mathrm{s,o}}$, and use correlations for the thermodynamic properties of moist air to determine $t_{\mathrm{a,o}}$.

In numerical integration, the tower was divided into 160 intervals with equal $\Delta\mathrm{NTU} = h_D a \Delta V / \dot{m}_{\mathrm{a}}$ across each interval. Above this number of intervals, numerical results were found to be unaffected by the increased intervals. Finite difference equations derived from Eqs. (7)–(9) were used to determine the values of h_a, W_{a} and t_{s} at each node between intervals or at the top or bottom position. In calculation,

Table 1
Comparison of calculation results with different methods

No.	Liquid	Given conditions						Numerical method			Simple ε-NTU method			Corrected results													
		$t_{s,i}$ (°C)	$\xi_{s,i}$ (%)	$t_{a,i}$ (°C)	$W_{a,i}$ (g_w/kg_a)	$\frac{\dot{m}_a}{\dot{m}_{s,i}}$	NTU	$t_{s,o}$ (°C)	$t_{a,o}$ (°C)	$W_{a,o}$ (g_w/kg_a)	$t^0_{s,o}$ (°C)	$t^0_{a,o}$ (°C)	$W^0_{a,o}$ (g_w/kg_a)	$t^{H(1)}_{s,o}$ (°C)	$t^{H(2)}_{s,o}$ (°C)	$t^C_{s,o}$ (°C)	$t_{s,o}$ (°C)	e_{t_s} [a] (%)	$t^{H(1)}_{a,o}$ (°C)	$t^{H(2)}_{a,o}$ (°C)	$t^C_{a,o}$ (°C)	$t_{a,o}$ (°C)	$W^{H(1)}_{a,o}$ (g_w/kg_a)	$W^{H(2)}_{a,o}$ (g_w/kg_a)	$W^C_{a,o}$ (g_w/kg_a)	$W_{a,o}$ (g_w/kg_a)	e_{W_a} [a] (%)
1.1	Water	55	–	35	21.55	1.0	3	30.47	43.29	59.7	32.49	41.74	54.57	−1.79	0.07	−0.01	30.76	1.18	0.77	0.6	0	43.11	3.06	1.97	0.01	59.62	0.21
1.2	Water	40	–	15	6.37	1.0	3	19.82	33.01	33.09	19.54	32.64	32.28	−0.12	0.33	−0.03	19.72	−0.50	0.08	0.32	0.02	33.06	0.32	0.56	0.04	33.22	−0.49
1.3	Water	40	–	35	21.55	1.0	3	29.76	35.96	38.49	29.64	35.66	37.86	−0.11	0.19	−0.01	29.71	−0.49	−0.06	0.24	0.01	35.84	0.2	0.5	0.01	38.58	−0.53
1.4	Water	40	–	35	21.55	0.5	3	33.12	38.16	43.58	32.84	38.05	43.25	0	0.29	0	33.1	−0.29	−0.09	0.13	0.01	38.09	0.05	0.3	0.01	43.61	−0.14
1.5	Water	40	–	35	21.55	2.0	3	28.26	32.95	32.36	28.76	32.62	31.51	−0.47	0.06	0	28.35	0.77	−0.03	0.22	0	32.81	0.4	0.42	0	32.34	0.19
1.6	Water	40	–	35	21.55	1.0	6	28.75	36.42	40.02	28.89	35.97	38.97	−0.32	0.1	−0.01	28.67	−0.71	0.18	0.29	0	36.45	0.48	0.67	0.01	40.13	−0.60
1.7	Water	40	–	35	21.55	1.0	1	32.67	36.02	33.58	32.42	35.9	33.32	−0.01	0.2	−0.01	32.66	−0.14	−0.04	0.11	0	35.97	0.03	0.25	0.01	33.61	−0.25
2.1	LiBr	15	50	35	21.55	0.6	3	31.41	18.82	4.19	31.64	18.96	4.48	0.23	−0.41	−0.01	31.45	−0.24	−0.07	-0.09	0.02	18.81	−0.29	−0.03	0.01	4.16	−0.17
2.2	LiBr	15	50	35	21.55	0.3	3	23.66	17.18	3.77	23.84	17.21	3.83	0.02	−0.19	0	23.66	0.00	−0.01	−0.03	0	17.18	−0.05	−0.01	0	3.77	0.00
2.3	LiBr	65	50	35	21.55	0.6	3	49.26	57.87	35.35	48.12	57.13	35.94	0.69	0.18	−0.02	48.98	−1.78	0.38	0.48	0.01	58	−1.1	0.86	0.02	35.72	−2.68
2.4	LiBr	65	50	35	21.55	0.3	3	54.53	61.1	41.93	53.77	60.91	42.15	0.31	0.47	−0.01	54.54	0.10	−0.05	0.27	0.01	61.14	−0.83	0.54	0.02	41.88	0.25
2.5	LiBr	65	50	35	21.55	0.3	1	58.18	52.44	34.81	57.76	52.33	34.71	0.05	0.35	0	58.16	−0.29	−0.01	0.13	0	52.45	−0.14	0.27	0.01	34.84	−0.23
2.6	LiBr	65	50	35	21.55	0.3	6	52.97	63.49	45.54	51.99	63.46	45.96	0.43	0.57	−0.01	52.98	0.08	−0.23	0.29	0.01	63.54	−1.09	0.5	0.03	44.49	4.38
3.1	LiCl	15	35	35	21.55	1.0	3	35.59	20.39	5.12	34.8	20.28	5.59	0.55	0.36	−0.04	35.66	−0.34	−0.08	0.12	0.06	20.37	−0.57	0.05	0.02	5.09	−0.18
3.2	LiCl	15	35	35	21.55	0.5	3	26.36	17.69	4.28	26.04	17.64	4.36	0.05	0.28	−0.01	26.37	−0.09	−0.02	0.05	0.01	17.68	0.1	0.01	0	4.28	0.00
3.3	LiCl	65	35	35	21.55	1.0	3	45.94	54.87	34.67	45.03	53.94	35.89	0.51	0	−0.02	45.52	−2.20	0.9	0.07	0.01	54.91	−0.93	0.12	0.02	35.1	−3.28
3.4	LiCl	65	35	35	21.55	0.5	3	50.64	59.27	43.76	49.87	59.18	45.13	0.67	0.08	−0.03	50.58	−0.42	0.04	0.07	0.02	59.32	−1.49	0.15	0.05	43.83	−0.32
3.5	LiCl	65	35	35	21.55	0.5	1	55.32	51.55	36.67	55.04	51.5	37.01	0.22	0.09	−0.01	55.34	0.21	0.01	0.04	0.01	51.56	−0.49	0.08	0.02	36.61	0.40
3.6	LiCl	65	35	35	21.55	0.5	6	48.61	61.44	47.25	47.46	61.56	49.34	0.97	0.06	−0.04	48.45	−0.98	−0.13	0.08	0.04	61.55	−2.07	0.17	0.08	47.51	−1.01
4.1	$CaCl_2$	15	40	35	21.55	1.0	3	35.64	21.66	7.39	34.9	21.49	8.11	0.86	0.06	−0.08	35.74	−0.48	0.05	0.03	0.08	21.64	−0.8	0.02	0.04	7.33	−0.42
4.2	$CaCl_2$	15	40	35	21.55	0.4	3	24.94	17.51	5.54	24.82	17.5	5.64	0.05	0.08	−0.01	24.95	−0.10	−0.02	0.02	0.02	17.51	−0.12	0.006	0.01	5.53	−0.06
4.3	$CaCl_2$	65	40	35	21.55	1.0	3	40.65	50.29	39.05	40.02	48.41	40.08	0.11	−0.01	−0.01	40.12	−2.18	1.5	0.13	0	50.04	−0.7	0.32	0.01	39.67	−3.54
4.4	$CaCl_2$	65	40	35	21.55	0.4	3	46.61	57.49	57.51	45.05	57.13	60.46	1.23	0.14	−0.06	46.36	−1.36	0.27	0.16	0.05	57.61	−3.07	0.47	0.14	58.00	−1.36
4.5	$CaCl_2$	65	40	35	21.55	0.4	1	52.34	50.73	46.63	51.71	50.61	47.51	0.47	0.17	−0.02	52.33	−0.08	0.04	0.08	0.02	50.75	−1.15	0.25	0.05	46.65	−0.08
4.6	$CaCl_2$	65	40	35	21.55	0.4	6	44.19	59.53	62.32	42.07	59.22	66.46	1.46	0.1	−0.08	43.55	−3.08	0.39	0.22	0.06	59.89	−3.65	0.6	0.16	63.59	−3.12

[a] Definition for relative errors: $e_x = (x_o^{an} - x_o^n)/(x_o^n - x_i)$, here x represents the variable discussed and superscripts 'an' and 'n' indicates results by analytical and numerical method, respectively.

concentrations of desiccant solutions were calculated according to Eq. (32). Properties of air in equilibrium with water or desiccant solutions were calculated from the properties of water or desiccant solutions [15–17]. Numerical integration was implemented starting from the air inlet side of the tower. Guess values of water or solution conditions at the position of air inlet must be assumed. Iteration proceeded until the calculated water or solution inlet conditions matched the actual values.

Twenty-five cases with three different desiccant solutions commonly used in practical applications were examined, with parameters $t_{s,i}$, $t_{a,i}$, $\frac{\dot{m}_a}{\dot{m}_{s,i}}$ and NTU varied from low to high values while holding the inlet concentration at a typical value for each desiccant solution for simplicity in presentation. Results were presented in Table 1. The comparison shows that the analytical results are generally satisfactory and the improvement over the simple ε-NTU method is significant. Relative errors by the analytical model are generally less than 2% except for the cases with high solution temperature and large air to solution mass flow rate ratio (such as cases 2.3, 3.3 and 4.3) or large NTU numbers (such as cases 2.6 and 4.6). For the cases presented in Table 1, the average of absolute relative errors for calculated outlet water or solution temperatures by the analytical method are 6.3 times less than that by the simple ε-NTU method and 4.63 times less for outlet air humidity ratio. Changing the inlet solution concentration did not result in any additional error between the two models. In cooling towers, the equilibrium conditions of air at water surface are along the 100% relative humidity line. In desiccant systems, the equilibrium conditions of air at solution surface are along different relative humidity lines approximately with different solution concentrations [18]. In Table 1, the typical values of inlet concentrations for different desiccant solutions will correspond to different equilibrium relative humidities and thus the validity of the analytical model for different inlet concentrations are also demonstrated in effect.

Corrections to the simple ε-NTU method for outlet water or solution temperatures, air temperatures and humidity ratios are also presented in Table 1. In presentation, $x^{H(1)}$ represents correction values due to the effects of nonlinearities of the humidity ratio and enthalpy of air in equilibrium with water or solution, $x^{H(2)}$ due to the effect of water loss by evaporation on water or solution energy balance and x^{C} due to the effect of variation of the heat capacity rate ratio C_s^*. It is found that x^{C} values are much smaller than the other correction values and can be neglected in calculations for simplicity. $x^{H(1)}$ values play the most important role in total correction values while $x^{H(2)}$ values play a significant role.

In conclusion, analytical expressions for the corrections to the simple effectiveness-NTU method are developed using perturbation technique for cooling towers and liquid desiccant–air contact systems. The developed model takes into consideration the effect of nonlinearities of humidity ratio and enthalpy of air in equilibrium with water or desiccant solutions. The model also takes into consideration the effect of water loss by evaporation and the effect of variation of the specific heat capacity of water or solution. The comparison with numerical integration of the dimensional heat and mass transfer equations shows that the analytical results are generally satisfactory and the improvement over the simple ε-NTU method is significant. In order to further reduce the errors by the analytical model for the cases with high solution temperature and large air to solution mass flow rate ratio or large NTU numbers, higher order perturbation method may be needed. This deserves further research in the future.

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